**Directional derivatives and gradients**

**APCalculus**

**1. Find** ∇*f* **if**

**(a)** *f* (*x*, *y*) = *– x*2*y* + *xy*2 + *xy* **(b)** *f* (*x*, *y*) = sin *x* cos *y*

Ans. **(a)** (*–*2*xy* + *y*2 + *y*,– *x*2 + 2*xy* + *x*)

**2. A function** *z* = *f* (*x*, *y*) **and a point** *P* **are given. Find the directional derivative of** *f* **in the indicated directions.**

**(a)** *f* (*x*, *y*) = , *P* = (1,1).

**i.** In the direction of *v* = (1, –1*⟩*.

**ii.** In the direction toward the point *Q* = (*–*2, *–*2).

**Ans. i.** 0 **ii.** 2/9

**(b)** *f* (*x*, *y*) = – 4*x* + 3*y*, *P* = (5,2).

**i.** In the direction of *v* = (3, 1).

**ii.** In the direction toward the point *Q* = (2,7).

**3. If**

**(a)** *f* (*x*, *y*) = *x*2 + 2*y*2 *– xy –* 7*x*, *P* = (4,1),

**(b)** *f* (*x*, *y*) = *x*2*y*3 *–* 2*x*, *P* = (1,1),**then**

**i. Find the direction of maximal increase of** *f* **at** *P***.**

**ii. What is the maximal value of** *Du f* **at** *P***?**

**iii. Find the direction of minimal increase of** *f* **at** *P***.**

**iv. Give a direction** *u* **such that** *Du f* = 0 **at** *P***.**

**Ans. i. (0,0) ii. 0 iii. (0,0)** **iv.** All directions give a directional derivative of 0.

**4. A function** *w* = *f* (*x*, *y*, *z*)**, a vector** *v* **and a point** *P* **are given.**

**i. Find** ∇*f* (*x*, *y*, *z*)**.**

**ii. Find the directional derivative of** *f* **at** *P* **in the direction Of** *v***.**

**(a)** *f* (*x; y; z*) = 3*x*2*z*3 + 4*xy –* 3*z*2, *v* = (1,1,1), *P* = (3,2,1)

**(b)** *f* (*x; y; z*) = sin(*x*) cos(*y*) *ez*, *v* = (2,2,1), *P* = (0,0;0)

**Ans. (a) i.**  (6*xz*3 + 4*y*,4*x*,9*x*2*z*2 –6*z*) **ii.** 113/

**Ragovsky**

**5.** Find the directional derivative of *f* (*x, y, z*)= *xy* + *z*3 at *P* = (3*,*−2*,*−1)in the direction pointing to the origin.

**6.** A bug located at (3*,* 9*,* 4)begins walking in a straight line toward (5*,* 7*,* 3). At what rate is the bug’s temperature changing if the temperature is *T* (*x, y, z*)= *xey*−*z*? Units are in meters and degrees Celsius.

**Ans.** *D***u***f (P)* = −*e*5/3 ≈ −49*.*47

**7.** The temperature at location (*x, y*)is *T* (*x, y*)= 20 + 0*.*1(*x*2 − *xy*) (degrees Celsius). Beginning at *(*200*,* 0*)* at time *t* = 0 (seconds), a bug travels along a circle of radius 200 cm centered at the origin, at a speed of 3 cm/s. How fast is the temperature changing at time *t* = *π/*3?

**8.** Suppose that ∇*fP* = (2*,*−4*,* 4). Is *f* increasing or decreasing at *P* in the direction **v** = (2*,* 1*,* 3)?

**Ans.** *f* is increasing at *P* in the direction of v.

**9.** Let *f* (*x, y, z*)= sin(*xy* + *z*)and *P* = (0*,*−1*, π*). Calculate *Du f (P)*, where *u*is a unit vector making an angle *θ* = 30° with ∇*fP* .

**Ans.** *D***u***f (P)* = √6/2

**10.** *Find an equation of the tangent plane to the surface at the given point.*

**(a)** *x*2 + 3*y*2 + 4*z*2 = 20, *P* = (2*,* 2*,* 1)

**(b)** *xz* + 2*x*2*y* + *y*2*z*3 = 11, *P* = (2*,* 1*,* 1)

Ans. (b)43

**11.** Find a unit vector in the direction in which *f* increases most rapidly at *P*, and find the rate of change of *f* at *P* in that direction.

**(a)** *f* (*x*, *y*) = 4*x*3*y*2; *P(*−1*,* 1*)*

**(b)** *f* (*x*, *y*) = 3*x* − ln *y*; *P*(2*,* 4)

**Ans.**

**(a)** 4 **(b)** /4

**12.** Find a unit vector in the direction in which *f* decreases most rapidly at *P*, and find the rate of change of *f* at *P* in that direction.

**(a)** *f* (*x*, *y*) = 20 − *x*2 − *y*2; *P*(−1*,*−3)

**(b)** *f* (*x*, *y*) = *exy* ; *P*(2*,* 3)

**Ans.**

**(a)** –2 **(b)** –*e*6

**13.** Find the directional derivative of *f* (*x*, *y*)= *x*2 + 4*y*2 at *P* = (3*,* 2)in the direction pointing to the origin.

**Ans.**

*Du f (*3*,* 2*)* = −50/

**14. find an equation of the tangent plane and find a set of symmetric equations for the normal line to the surface at the given point.**

**(a)** *z* = *x*2 – *y*2, (3, 2, 5)

**(b)** *xy* – *z* = 0, (– 2, – 3, 6)

**(c)** *xyz* = 10, (1, 2, 5)

**(d)** *z* = *ye*2*xy*, (0, 2, 2)

**(e)** *y* ln(*xz*2) = 2, (*e*, 2, 1)

**Ans.**

**(a)** 6x – 4y – z = 5 = =

**(b)** 3*x* + 2*y* + *z* = 5 = =

**(c)** 10*x* + 5*y* – *z* = 0 = =

**(d)** 8*x* + *y* + *z* = 5 = =

**(e)** *x* + *y* + 4*z* = 5 = =